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**QUEEN'S UNIVERSITY FINAL EXAMINATION**
FACULTY OF ARTS AND SCIENCE
SCHOOL OF COMPUTING

CISC 102 001
Dr. Robin Dawes
June 21, 2019

**INSTRUCTIONS TO STUDENTS:**

This examination is 3 HOURS in length.

There are 10 questions.

Please answer all questions on this exam paper.

No aids are allowed.

Put your student number on all pages of this exam paper, including the front.

GOOD LUCK!

**PLEASE NOTE:**
**Proctors are unable to respond to queries about the interpretation of exam questions.**
**Do your best to answer exam questions as written.**



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**HAND IN**
Answers recorded on exam paper

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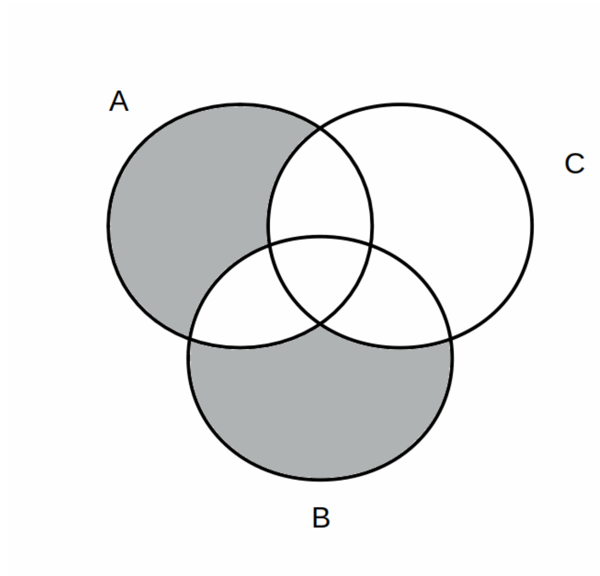
## QUESTION 1: [10 marks]

Consider this Venn Diagram




Express the shaded area as a set, using intersections, unions and/or complements of A, B and C

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## QUESTION 2: [10 marks]

Consider the statements S1: Some of my windows are clean.

S2: All things that are worthless are not clean.

(a) Which of these statements can be concluded from S1 and S2?

S3: None of my windows are worthless.

S4: All of my worthless things are windows.

S5: I have at least one window that is not worthless.

S6: I have at least one worthless window.

(b) State the negation of the statement you selected in (a) Do not simply write “It is not true that ...”

## QUESTION 3: [10 marks]

Prove the following:

For every integer $n$, if $n^3 + 5n^2 - n + 6$ is even, then $n$ is even

Use Proof by Contradiction.

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## QUESTION 4: [10 marks]

Let X = {1,2,3,4,5} and let Y = {a,b,c,d,e,f}

(a) How many 1-to-1 functions are there from X to Y? Explain your answer.

(b) How many 1-to-1 functions are there from Y to X? Explain your answer.

(c) How many onto functions are there from X to X? Explain your answer.

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**QUESTION 5: [10 marks]**

Use induction to prove

$$1 + 3 + 5 + \ldots (2n - 1) = n^2, \quad \forall n \in \mathbb{N}, n \geq 1$$

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**QUESTION 6: [10 marks]**

Prove the following:

$$\forall\ n\ \geq 1\ ,\ gcd(n, n+3) \leq 3$$

**QUESTION 7: [10 marks]**

Let $R$ be the relation on $\mathbb{Z}$ defined by

$$(a,b) \in R \text{ iff } |a-b| \leq 3$$

For example, $(3,5) \in R$ because $|3-5| = 2$

$(7,2) \notin R$ because $|7-2| = 5$

(a) Prove or disprove: R is reflexive

(b) Prove or disprove: R is symmetric

(Question continues on the next page)

(c) Prove or disprove: R is transitive

(d) Is R an equivalence relation? Explain your answer.

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## QUESTION 8: [10 marks]

Given that $\binom{30}{8} = 5852925$ and $\binom{29}{8} = 4292145$

determine the value of $\binom{29}{7}$

Explain your reasoning.

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## QUESTION 9: [10 marks]

Three bull-whip cracking archaeologists named Alex, Billy and Chris have joined forces to steal 12 unique treasures. Now they are deciding how to divide up the loot.

In how many different ways can the archaeologists distribute the 12 treasures amongst themselves, with the restriction that each archaeologist gets at least one treasure? Explain your answer.

Your answer may include factorials, exponentials and combinatorial notation such as $\binom{100}{23}$ if you wish. You are not required to work out the final value (it is very large!)

(Hint: one approach is to compute all possible distributions, then remove the ones in which some archaeologist gets none of the treasures – be careful!)

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## QUESTION 10: [10 marks]

All 13 members of the Karcrashian family have shown up for the annual family photograph. The family's 3 pet sheep are also going to be in the photo – the sheep are all clones so they are identical. The people are all distinct.

How many different-looking ways can the whole gang (people and sheep) line up in a straight line? Explain your answer.

Your answer may include factorials, exponentials and combinatorial notation such as $\binom{100}{23}$ if you wish. You are not required to work out the final value (it is very large!)

***This is the last question on the examination.***

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This page can be used for rough work.

Enjoy the rest of your summer!