CISC-203*
Test #1
September 25, 2018

Student Number (Required) ______________________

Name (Optional) ________________________________

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

The test will be marked out of 50.

Question 1	/10
Question 2	/10
Question 3	/10
Question 4	/10
Question 5	/10
TOTAL	/50

By writing my initials in this box, I authorize the disposal of this test paper if I have not picked it up by January 15, 2019.

Question 1 : (10 marks)

Let $f : A \to B$ and $g : B \to C$ be functions, where A, B and C are finite sets.

Prove the following statement:

If f and g are both ***onto,*** then $|A| \geq |C|$

Use any valid proof technique:

Question 2 : (10 marks)

Prove the following using any valid proof technique:

$$\sum_{i=1}^{n}(2i-1) = n^2 \qquad \forall \text{ integers } n \geq 1$$

Question 3 : (10 marks)

(a) [3 marks] Find a value of k that makes the following statement true:

All integers $n \geq k$ can be written as $n = 3a + 5b$ where a and b are positive integers.

(b) [7 marks] Use either **Proof by Minimum Counter-Example** or **Proof By Induction** to prove that your value of k makes the statement true.

Question 4 : (10 Marks)

Let $\pi, \sigma,$ and θ be permutations in S_n

(a) [5 marks]

Prove that π^{-1} is unique. That is, prove that

if $\pi \circ \sigma = \sigma \circ \pi = \iota$ and $\pi \circ \theta = \theta \circ \pi = \iota$, then $\sigma = \theta$

(b) [5 marks]

We use π^x to represent $\underbrace{\pi \circ \pi \circ \cdots \circ \pi}_{x}$

For example, $\pi^3 = \pi \circ \pi \circ \pi$

Prove $\forall x \geq 1, \quad \pi^x = \pi^{x+1} \Rightarrow \pi = \iota$

Question 5 : (10 Marks)

We call a permutation π a ***derangement*** if $\pi(i) \neq i \;\; \forall \; i \in \{1, 2, \ldots, n\}$

For example

$\pi = [4\ 5\ 6\ 7\ 1\ 2\ 3]$ is a derangement

$\tau = [4\ 2\ 6\ 7\ 1\ 3\ 5]$ is not a derangement because $\tau(2) = 2$

(a) [5 marks]

Let π be a permutation of $\{1, 2, \ldots, n\}$ where $n \geq 2$

Prove : if π is **not** a derangement, the cycle representation of π **must** contain at least two cycles. Use any valid proof technique.

(b) [5 marks]

Prove or disprove this statement:

If π is a derangement, then π^{-1} is also a derangement

This page is for rough work or overflow.