CISC-203*
Test #4
November 15, 2018

Student Number (Required) ______________________

Name (Optional) ________________________________

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

The test will be marked out of 50.

Question 1	/10
Question 2	/15
Question 3	/10
Question 4	/10
Question 5	/5
TOTAL	/50

By writing my initials in this box, I authorize Dr. Dawes to destroy this test paper if I have not picked it up by January 15, 2019.

Question 1: (10 Marks)

Define $\preceq$ on CISC courses by

$x \preceq y$ **iff** $x = y$ **or** x **is a prerequisite of** y

For example $203 \preceq 203$, $121 \preceq 124$, **etc**

Let S be the following (somewhat reduced) set of CISC courses, and let R be the following set of $\preceq$ connections.

S = { 102, 121, 124, 203, 204, 235 }

R = { (102, 203), (102, 204), (121,124), (121,203), (121, 204), (124, 235), (203, 235) }

(a) [6 marks] List all the pairs that need to be added to R to make (S, R) a poset

(b) [4 marks] Determine the height and width of the poset

Question 2 : (15 marks)

Let $S = \{1, 2, 3, 4\}$ and let F be the set of all functions from S to S.

Note that F contains functions which are bijections as well as functions that are not.

We define the relation $\preceq$ on F as follows:

Let f and g be functions in F. $\quad f \preceq g$ **iff** $f(i) \leq g(i) \quad \forall\, i \in S$

(where $\leq$ has the standard "less-than-or-equal" meaning)

For example, if f and g are given by this table, then $f \preceq g$

i	$f(i)$	$g(i)$
1	2	3
2	3	3
3	1	2
4	2	4

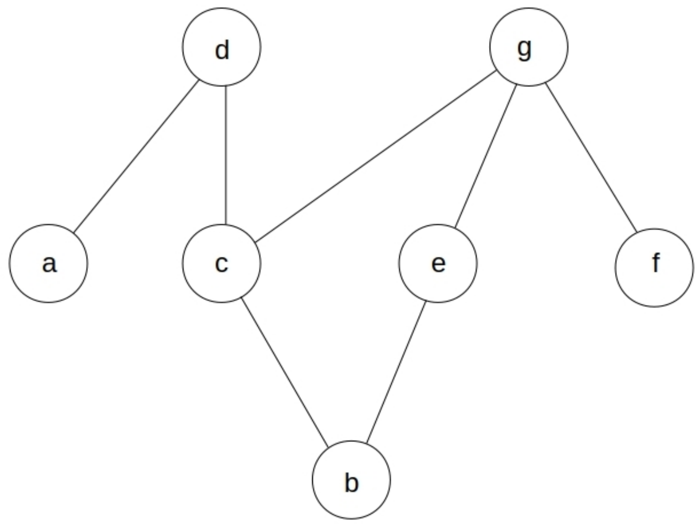
(a) [3 marks] Find and show two functions f and g such that neither $f \preceq g$ nor $g \preceq f$ is true. (That is, find two incomparable functions.)

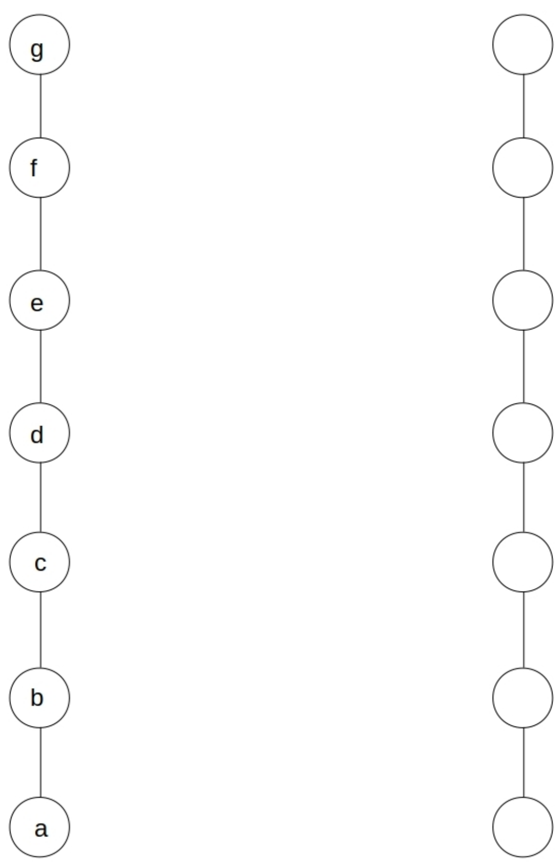
(b) [9 marks] Prove that $P = (F, \preceq)$ is a poset.

(c) [3 marks] Either find the minimum element of P, or explain why P does not have a minimum element.

Question 3: (10 Marks)

Here is a Hasse Diagram representing a poset. Also given is a linear extension of the poset. Find and show another linear extension such that the two linear extensions form a realizer for the poset.





Question 4 : (10 Marks)

Let $P = (X, \preceq)$ and $Q = (Y, \preceq^*)$ be two posets, where X and Y are finite sets, and $\preceq$ and $\preceq^*$ represent two different relations.

Suppose $X \cap Y \neq \emptyset$. Prove that $R = (X \cap Y, \preceq \cap \preceq^*)$ is a poset.

(Recall that a relation is a set of ordered pairs, and the intersection of two relations is the set of ordered pairs that belong to both relations.)

Question 5 : (5 Marks)

In this question we will use the expression $a \preceq b$ **where** a **and** b **are positive integers to mean "a divides b"**

For example, $3 \preceq 3$ **,** $13 \preceq 26$ **,** $4 \preceq 12$ **etc**

Let S = {2, 3, 4, 8, 24}, and let P = (S, $\preceq$**) You do not have to prove that P is a poset.**

Let T = { {1} , {1,2} , {3} , {1,2,3} , {1,2,3,4,5} , {5} }, and let Q = (T, $\subseteq$**) You do not have to prove that Q is a poset.**

Show that P and Q are isomorphic. (Hint: posets are isomorphic if they have the same Hasse Diagram.)

NOTE: THIS QUESTION CONTAINS AN ERROR!!!!

S should be defined as S = {2,3,4,5,12,60}