CISC-203*
Test #5
November 29, 2018

Student Number (Required) ______________________

Name (Optional) ________________________________

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

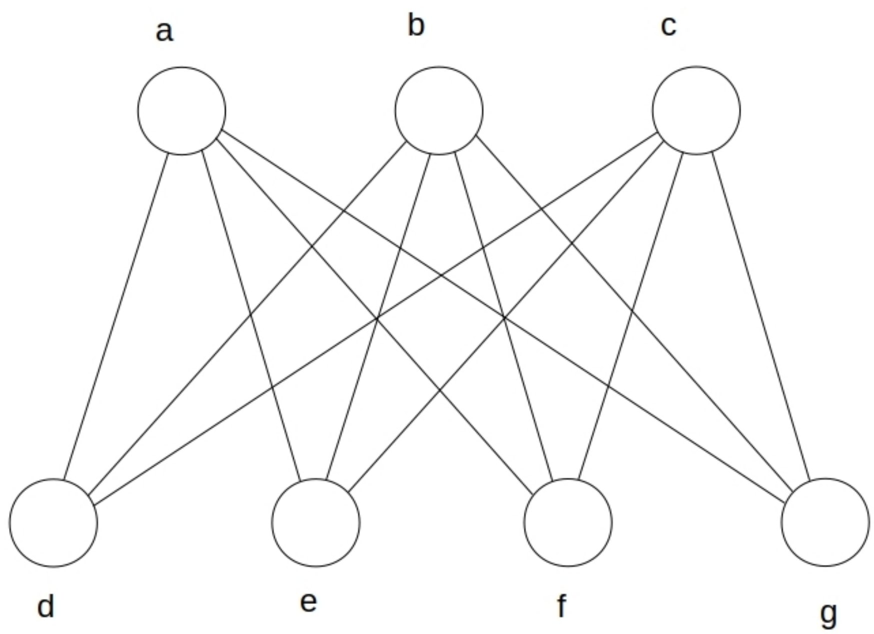
Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

The test will be marked out of 50.

Question 1	/10
Question 2	/15
Question 3	/10
Question 4	/10
Question 5	/5
TOTAL	/50

☐ By writing my initials in this box, I authorize Dr. Dawes to destroy this test paper if I have not picked it up by January 15, 2019.

Question 1: (10 Marks)



Does this graph contain a cycle that includes all 7 vertices? If so, list the vertices in the order they appear in the cycle. If not, explain why there is no such cycle.

Question 2 : (15 marks)

Let $S = \{1, 2, 3\}$ and let F be the set of all functions from S to S.

Note that F contains functions which are bijections as well as functions that are not.

We can create a graph in which *each vertex represents a function in F*. The edges of the graph are determined as follows:

The vertex for function f is adjacent to the vertex for function g if and only if there is exactly one difference between the functions.

More formally, the vertices representing functions f and g are adjacent in the graph if and only if there is exactly one $i \in S$ for which $f(i) \neq g(i)$

For example, consider the functions f, g and h shown in this table

i	$f(i)$	$g(i)$	$h(i)$
1	2	1	2
2	3	3	1
3	2	2	1

The vertices representing f and g are adjacent because $f(1) \neq g(1)$ and there is no other i such that $f(i) \neq g(i)$. The vertices representing f and h are not adjacent because there are two i such that $f(i) \neq h(i)$. The vertices representing g and h are not adjacent because $g(i) \neq h(i) \quad \forall\ i$

Question continues on the next page

(a) [5 marks] Let f be the function from the example above. List ALL the functions whose vertices are adjacent to the vertex for function f

(b) [5 marks] Show that this graph is connected. You are not required to draw the graph (but you may if you want to). Hint: show there is a path from every function to the function p, where $p(1) = p(2) = p(3) = 1$

(c) [5 marks] Is this graph Eulerian? Explain your answer. You may use the fact that the graph is connected, even if you did not answer part (b)

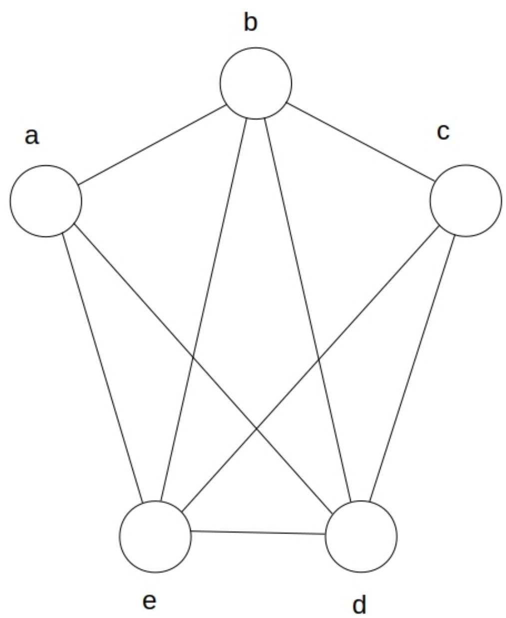
Question 3: (10 Marks)

Let G be a connected graph on ≥ 3 vertices. Let P_1 and P_2 be paths in G. We say P_1 is *contained in* P_2 if all edges in P_1 are also in P_2

Show that *"contained in" is* a partial ordering of the set of all paths in G.

Question 4 : (10 Marks)

Consider the graph that results from deleting one edge from K_5 :



(a) [7 marks]If this graph is planar, show a planar drawing of it. If it is not planar, explain why not.

(b) [3 marks] What is the colouring number of this graph? Explain.

Question 5 : (5 Marks)

Let G, H and J be graphs with $G \cong H$ and $H \cong J$.

(Recall that $\cong$ means "is isomorphic to")

Either prove that $G \cong J$ or give a counterexample.