Question 1:

$(p \rightarrow q) \rightarrow (r \rightarrow s)$

Using a truth table

p	q	r	s	$p \rightarrow q$	$r \rightarrow s$	$(p \rightarrow q) \rightarrow (r \rightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	T	F	F	F	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	T	T
F	F	F	F	T	T	T

Construct an eliminating disjunction for each of the highlighted lines, and combine them in a conjunction.

(not p or not q or not r or s) and (p or not q or not r or s) and (p or q or not r or s)

Marking: if they know how to do it but get it all wrong, they should get at least half the marks. Similarly if they try to convert directly to CNF (replacing implications with ors, etc) but they get all tied up, they should get at least half the marks. If they have a small error in their truth table but they build a CNF formula that matches their ***truth table, give them about 17/20. If their truth table is correct but they don't know how to build a proper CNF formula from it, they should get about 13/20***

Question 2:

Φ : not ((p or (p $\rightarrow$ q)) and (r or (r $\rightarrow$ s)))

thus not Φ : ((p or (p $\rightarrow$ q)) and (r or (r $\rightarrow$ s)))

We can show not Φ is valid as follows:

not Φ : ((p or (not p or q)) and (r or (not r or s)))
(using provable equivalence)
not Φ : ((p or not p or q) and (r or not r or s))

not Φ is now in CNF – it is valid iff each of its clauses is valid

The first clause is valid since it contains both "p" and "not p". The second clause is valid since it contains both "r" and "not r". Thus not Φ is valid, which means (by the theorem on the information sheet) that Φ is not satisfiable.

Marking: As before, if they knew what to do but couldn't do it, they should get about half the marks.

Some of the students may have tried to use completely different methods – use your judgment as to whether their answers are acceptable, always bearing in mind that a student who makes a sensible attempt should get about half the marks even if they cannot solve the problem.

Question 3:

Predicates:

I(x) : x is an intellectual
W(x) : x writes letters to the editor
S(x) : x is a student
R(x) : x is a reader
L(x) : x is a letter to the editor

Not all intellectuals write letters to the editor

TE x (I(x) and not W(x))

If there are any letters to the editor, then all students are intellectuals

TE x (L(x)) implies FA x (S(x) implies I(x))

If there are no letters to the editor, then there are no readers

not TE x (L(x)) implies not TE x (R(x))

If there are any readers and any students, then there is at least one intellectual

(TE x (R(x)) and TE x (S(x))) implies TE x (I(x))

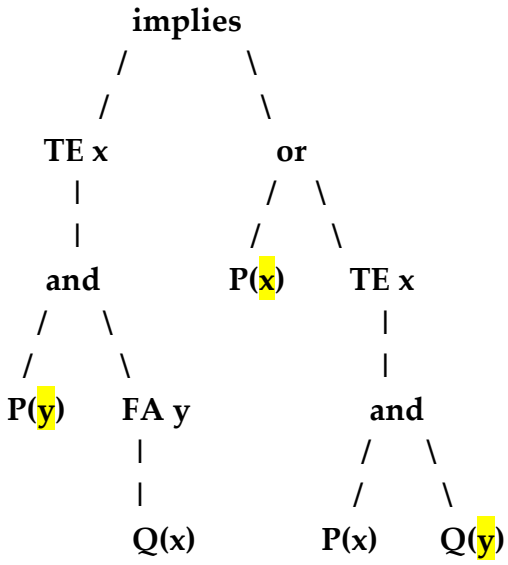
Anyone who writes a letter to the editor is either a reader or a student, but not both

FA x (W(x) implies ((R(x) or S(x)) and not (R(x) and S(x))))

Marking: Defining useful predicates : 5 marks. Their predicates don't have to be the same as mine.

Translating the statements: 2 marks each. My answers are not the only correct solutions. If an answer is partially correct, give 1 mark for that answer.

Question 4



The variables highlighted in yellow are free. The others are bound.

Marking: Tree: 2 marks Free variables: 1 mark each

Note that some students may show the variables as leaves of the tree, putting them below the predicates. That is fine (it is how the textbook does it).

Partial marks as you feel appropriate.