CISC-204*
Test #2
February 13, 2009

Student Number (Required) _____

Name (Optional)_____

This is a closed book test. You may not refer to any resources other than the information sheet at the back of the test. You may remove the information sheet.

This is a 50 minute test. No-one will be permitted to leave during the last ten minutes of the test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

Academic dishonesty will not be tolerated.

The test will be marked out of 50.

Question 1	/8
Question 2	/12
Question 3	/14
Question 4	/16
TOTAL	/50

QUESTION 1: 8 Marks

Consider the following truth table for a formula Φ which contains atomic propositions p, q, r, and s. Express Φ in Conjunctive Normal Form.

p	q	r	s	Φ
T	T	T	T	F
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	F
T	F	F	T	F
T	F	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	F
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

QUESTION 2: 12 Marks

Using the predicates, function and constant given here, translate each of the following statements into predicate formulas.

  P(x) : x is a pillow
  C(x) : x is a cat
  A(x) : x is an athlete
  O(x,y) : x owns y
  i(x) : x's income
  J : Jeremiah

(a)   **(4 marks) There is an athlete's cat who owns every pillow.**

(b)   **(4 marks) All athletes who own no cats, have incomes equal to $1000.**

(c)   **(4 marks) If Jeremiah owns a cat, there is an athlete with the same income as Jeremiah.**

QUESTION 3: 14 Marks

Prove the following sequents

(a) (4 marks) $\vdash \forall x (P(x) \rightarrow P(x))$

(b) (10 marks) $\forall x (P(x) \rightarrow \forall y \, Q(y)), \; \exists y \, \neg Q(y) \; \vdash \; \forall x \, \neg P(x)$

QUESTION 4: 16 Marks

Consider the following predicate formula

$$\forall y \; (P(f(z),y) \rightarrow Q(f(z))) \land (\exists z \; R(z,z) \rightarrow \forall x \; P(x,z))$$

(a)   (6 marks) Draw the parse tree for the formula. Indicate which of the variables are free and which are bound.

(b)   (5 marks) Suppose the formula shown above is the result of evaluating $\Phi[t/x]$ for some formula Φ in which x occurs as a free variable, and some term t, where t is free for x in Φ. Give a possible definition of both Φ and t.

(c)   (5 marks) Suppose g(.,.,.) is a function that takes three parameters. Is g(z,y,x) free for z in the formula shown above? Why or why not?