

CISC-204*
Test #3
March 5, 2009

Student Number (Required) _____

Name (Optional) _____

This is a closed book test. You may not refer to any resources other than the information sheet at the back of the test. You may remove the information sheet.

This is a 50 minute test. No-one will be permitted to leave during the last ten minutes of the test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

Academic dishonesty will not be tolerated.

The test will be marked out of 50.

Question 1	/8
Question 2	/16
Question 3	/12
Question 4	/14
TOTAL	/50

QUESTION 1. 8 MARKS

Prove the validity of the following sequent in predicate logic

$$\exists x P(x) \vdash \neg (\forall x (\neg P(x)))$$

QUESTION 2: 14 MARKS

Consider the following argument:

If there are any tax payers, then all politicians are tax payers.

If there are any intellectuals, then all tax payers are intellectuals.

Therefore, if there are any tax-paying intellectuals, then all politicians are intellectuals.

Predicates:

$T(x)$: x is a tax payer

$P(x)$: x is a politician

$I(x)$: x is an intellectual

- (a) **(6 MARKS)** Translate the argument into a sequent in predicate logic, using the given predicates. (You will probably not need all the space on this page for this.)

(Question continues on next page)

(b) (10 MARKS) Prove that the sequent is valid.

(c) QUESTION 3: 12 MARKS

Let Φ be the formula

$$\forall x \forall y \exists z (P(x,y) \rightarrow P(y,z))$$

(a) **(6 MARKS)** Let M be the model:

$$A = \{a,b,c,d\}$$

$$R = \{ (b,c), (b,b), (b,a) \}$$

Does M satisfy Φ ? Justify your answer.

(b) **(6 MARKS)** Let M' be the model:

$$A = \{a,b,c\}$$

$$R = \{ (b,c), (a,b), (c,b) \}$$

Does M' satisfy Φ ? Justify your answer.

QUESTION 4: 14 MARKS

For each of the following sets of formulae, show that the set is consistent or prove that it is not consistent.

(a) **(7 MARKS)**

1. $\exists x \forall y ((P(x) \wedge P(y)) \rightarrow P(f(x,y)))$
2. $\neg \forall x (f(x,x) = x)$
3. $\exists x P(x)$

(b) **(7 MARKS)**

1. $\forall x (P(x) \rightarrow Q(x))$
2. $\exists x P(x)$
3. $\forall x \neg Q(x)$

BONUS QUESTION (0 MARKS)

Three farmers are disputing who owns a certain mule. It certainly belongs to one of them, but the owner may not want to admit it. Each farmer makes two statements, one of which is true and one of which is false.

Farmer A: It is not my mule. I had pizza for lunch.

Farmer B: It is C's mule. It is not my mule.

Farmer C: My wife's name is Olive. I own the mule.

Who owns the mule?

