CISC-204*
Test #5
April 3, 2009

Student Number (Required) ______________________

Name (Optional)________________________________

This is a closed book test. You may not refer to any resources other than the information sheet at the back of the test. You may remove the information sheet.

This is a 50 minute test. No-one will be permitted to leave during the last ten minutes of the test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

Academic dishonesty will not be tolerated.

The test will be marked out of 50.

Question 1	/10
Question 2	/15
Question 3	/15
Question 4	/10
TOTAL	/45

Question 1: 10 marks

Let c be a real number. We can define a fuzzy "less than or equal to" relationship by letting the truth value of "$x \leq c$" as follows:

$T(x \leq c)$	$= 1$	if $x \leq c$ (using the normal definition of $\leq$)
	$= (1 + c - x)$	if $c < x < c+1$
	$= 0$	otherwise

For example, here is a sketch of the graph of "x <= 5"

Using this definition, we can define a fuzzy "greater than" relationship as follows:

$$T(x > c) = 1 - T(x \leq c)$$

Draw a graph of $T(\,(\,(x \leq 10) \vee (x > 10)\,) \wedge (x \leq 12)\,)$

Question 2: 15 Marks

Recall that fuzzy set theory has this definition of the truth value of the statement "A is a fuzzy subset of B":

$$T(A \subseteq B) \equiv \frac{|A \cap B|}{|A|}$$

Given a universe of discrete objects and a fuzzy set S, we define $|S| = \Sigma\, S(x)$ (i.e. the sum of the membership values of all objects in S)

In crisp set theory, two sets are equal iff they are subsets of each other. We can use this idea to define a truth function for fuzzy set equality. Let X and Y be fuzzy sets on the same universe.

$$T(X = Y) \equiv T(X \subseteq Y \wedge Y \subseteq X)$$

Consider a universe of just 3 objects, with membership values in three sets as given in the following table.

	Vernon	Malini	Elmsley
Set A	0.2	0.8	0.5
Set B	0.7	0.5	0.8
Set C	0.4	0.2	0.9

Using the membership functions shown above, determine which pair of sets is most equal.

Question 3: 15 marks

We looked at several definitions for the "implies" operator "→" in fuzzy logic. One was based on the provable equivalence from propositional logic

$$p \rightarrow q \qquad \equiv \qquad \neg p \;\vee\; q$$

Another provable equivalence from propositional logic is

$$p \rightarrow q \qquad \equiv \qquad \neg p \;\vee\; (p \wedge q)$$

Suppose p and q are propositions which can have truth values in the set {0, 0.2, 0.5, 0.7, 1}

Draw tables showing the possible truth values of p → q using each of these two definitions.

Based on your tables, can you draw any conclusions about transferring provable equivalences from propositional logic to fuzzy logic? Justify your answer.

Question 4: 10 marks

Suppose we are creating a controller for an air-conditioner. The system has one sensor (Temperature) and two settable controls (Heat and Fan Speed). The rules are as follows:

R1. If Temperature is Very High, set Heat to Very Low and Fan Speed to High.

R2. If Temperature is High, set Heat to Low and Fan Speed to Medium.

R3. If Temperature is Medium, set Heat to Medium and Fan Speed to Low.

R4. If Temperature is Low, set Heat to High and Fan Speed to Medium.

(a) Sketch appropriate membership functions for this controller (use your own criteria for defining High, Low, etc.)

(b) Suppose at some point the measured Temperature has membership 0.4 in Low and membership 0.6 in Medium, and membership 0 in all the other sets. Describe in words what you would expect as resulting settings for Heat and Fan Speed, and explain your answer. (You are not required to compute actual values.)

Standard Fuzzy Logic Definitions:

Let p and q be propositions, with truth values T(p) and T(q)

$T(p \wedge q) = \min(T(p), T(q))$

$T(p \vee q) = \max(T(p),T(q))$

$T(\neg p) = 1 - T(p)$

Bonus Question: (0 Marks)

The fiendish logic professor says "Here I have a penny and a quarter. You get to make one statement. If you make a true statement I will give you one of the coins, but if you make a false statement I will give you nothing."

What statement should the student make in order to logically force the professor to give her the quarter?

(This problem is adapted from the works of Raymond Smullyan.)

Enjoy your summer, and I hope to see you in the fall.