CISC-365*
Test #3
February 12, 2019

Student Number (Required) ______________________

Name (Optional)________________________________

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, **but will not be re-marked under any circumstances.**

The test will be marked out of 50.

Question 1	/28
Question 2	/20
Question 3	/2
TOTAL	/50

Question 1 (28 marks)

Congratulations! Your international prestige as a problem-solver has earned you a new job – you now operate a guided-tour business in Balatronia.

Tourists sign up for 1-week (Short) or 2-week (Long) guided tours of the local mud pits during the summer season. There is a Short tour and a Long tour starting each week except the last week of the summer - in which there is only a Short tour. Each tour is worth a different amount of tip money, based on the wealth of the tourists. Your goal is to decide which tours to guide personally, without choosing any overlapping tours.

For example, suppose the summer season is 5 weeks long. The tours starting in each week might look like this. Tours are numbered according to the week in which they start.

<table>
<tr><th></th><th>Week 1</th><th>Week 2</th><th>Week 3</th><th>Week 4</th><th>Week 5</th></tr>
<tr><td>1-week tours</td><td>$Short_1$
Value = 10</td><td>$Short_2$
Value = 7</td><td>$Short_3$
Value = 12</td><td>$Short_4$
Value = 4</td><td>$Short_5$
Value = 9</td></tr>
<tr><td rowspan="2">2-week tours</td><td colspan="2">$Long_1$
Value = 20</td><td colspan="2">$Long_3$
Value = 18</td><td></td></tr>
<tr><td></td><td colspan="2">$Long_2$
Value = 22</td><td colspan="2">$Long_4$
Value = 16</td></tr>
</table>

One solution is to choose $Short_1, Long_2, Short_4, Short_5$ with a total value of 45

A better solution is to choose $Long_1, Short_3, Long_4$ with a total value of 48

In Week 1, you can guide either the 1-week tour ($Short_1$) or the 2-week tour ($Long_1$). In Week 2, you are either halfway through tour $Long_1$ or you can start guiding either of the tours that start in Week 2 (if you chose $Short_1$ in Week 1).

This question asks you to construct a Dynamic Programming solution to maximize your personal profit. Your solution must work on all instances, not just the example shown here.

(a) (5 marks) Explain how this problem satisfies the Principle of Optimality . Your explanation must be clear but a rigourous proof is not required.

(Hint: Suppose the optimal solution contains a particular tour X. What can you say about the chosen tours that precede X, and the chosen tours that follow X?)

NOTE to 2019F CMPE and CISC 365 students: We DID NOT discuss the Principle of Optimality this year. This topic will not be on our Test 3. The rest of this question is fine though!

(b) (8 marks) Give a recurrence relation for this problem.

Hint: Suppose the season is n weeks long. At the end of Week n, you will either be finishing $Short_n$ (and getting its value) or finishing $Long_{n-1}$ (and getting its value). Associate each of these possibilities with the appropriate subproblem. You may want to use "$P(k)$" to represent the maximum profit you can get in the first k weeks of the season.

(c) (5 marks) Explain and justify the order in which you will compute solutions to subproblems. If you plan to use a table to store solutions to subproblems, this is the place to describe it.

(d) (5 marks) Explain how you will determine the details of the optimal solution.

(e) (5 marks) What is the complexity of your algorithm? (Use n to represent the number of weeks in the summer season)

QUESTION 2 (20 Marks)

You and your worst enemy are playing a game. Between you are three piles of coins, containing n_1, n_2 and n_3 coins respectively. You take turns removing coins according to this rule: on your turn you must remove a positive number of coins from any one of the piles (ie you must take at least 1 coin). **You win the game if you take the very last coin.**

Each possible game situation is described by the sizes of the piles such as (4,7,2) or (2019,3,12)

If a single move can get from (a, b, c) to (d, e, f) we call (d, e, f) a ***child*** of (a, b, c). For example, we can get from $(8, 7, 5)$ to $(8, 4, 5)$ by removing 3 coins from the centre pile so $(8, 4, 5)$ is a child of $(8, 7, 5)$

We can label a game situation "W" if the player who takes the next turn can be sure of winning, and "L" if they can't. For example $(0, 0, 5)$ is a "W" situation – the player can take the whole third pile, but $(1, 1, 0)$ is an "L" - the player must take 1 coin, then the other player takes the last coin and wins.

In general, a situation is "W" if ***any*** of its children is labelled "L", and a situation is "L" if ***all*** of its children are labelled "W"

Create a recurrence relation to determine if situation (n_1, n_2, n_3) is a "W" or "L"

(Write your answer on the next page)

(a) (10 marks) Recursive part:

(b) (10 marks) Base case(s):

QUESTION 3 (2 Marks)

True or False:

... it was a typical TF as seen on previous tests ...